Robustness of structures: role of graph complexity

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Summary

Graphs are widely employed in many fields of Science and Humanities studies, e.g. network analysis, communications, or literature. This abstract representation serves to analyse large systems, in which each element plays a role in the overall behaviour. In this paper, this concept is applied to structural engineering. A scheme with beams (or columns) and joints can be represented, mathematically, as a set of nodes and edges. The information content of a graph, i.e. of the corresponding structure, is used as a measure of complexity by means of the so called Shannon's entropy. The deformation work plays a fundamental role in the description of the behaviour of the loaded scheme. Further, an optimization of the complexity lets to stress the capabilities of the novel metrics in the design of robust structures.

Keywords: graph theory; information theory, complexity, robustness.

1. Introduction

A graph is defined as a set of nodes and edges connecting the nodes. For their graphical representation and their topological properties, graphs are used in many fields of knowledge, even in those cases in which Mathematics seems not to be involved: a conceptual maps is a common tool for organizing and representing knowledge [1], the dictionary of synonyms and antonyms can be imagined as a set of connected lexical elements [2]. The human language shows some sort of organization and scale-free graph properties, reason for which communication between people speaking different languages is possible [3]. In Biology, metabolic networks show topological properties and exhibit a robust response towards internal defects and environmental fluctuations [4].

In applied sciences, graph representation serves for describing and studying complex systems. Communication networks and power supply can naturally be sketched as a set of nodes and connectors and robustness properties of such objects can be highlighted. Systems like the World-Wide Web can be seen as scale-free networks, i.e. systems in which the probability $P(k)$ that a node in the network is linked with k other nodes decreases with a power-law $P(k) \sim k^{-\gamma}$. They exhibit robustness towards random failures but no survivability in case of a targeted attack [5]. On the contrary, public transport systems present some of the characteristics of small-world networks [6]. To improve their robustness, i.e. not create delays in case of maintenance works, Lu and Shi [7] recommended to identify the most important hubs and links by considering (i) the degree of nodes and (ii) the edges weights.

In structural engineering, Henderson and Bickley [8] used graph theoretical approaches for the evaluation of the topological conditions that govern the degree of statical indeterminacy of a structure. Kron [9] made an analogy between electrical networks and elastic structures. Langefors [10] proposed a complete topological approach to structural mechanics. Various studies were being conduced on the topic throughout the second half of the Twentieth century. Recently, a large contribution has been given by Kaveh, which improved the computation techniques in matrix structural analysis ([11,12] and the references reported herein). From the available bibliography, it seems that very few works have dealt with topological properties of graphs applied to the behaviour

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